Translation:Attempt of a Theory of Electrical and Optical Phenomena in Moving Bodies/Section IV

1 The equations of motion of light for ponderable bodies.

1.1 Equations for the aether enclosed in ponderable bodies.

§ 39. Let us now turn to the motion of light in ponderable, dielectric, and completely transparent bodies. It shall be assumed, that they are moving with velocity $\mathfrak{p}$ in an arbitrary direction, and that, as already said, the molecules contain ions that are connected with certain equilibrium position.

For one of these particles we again denote the charge by *e*, and the displacement from the equilibrium position by $\mathfrak{q}$. The components $\mathfrak{q}_x$, $\mathfrak{q}_y$, $\mathfrak{q}_z$, as well as the velocities $\dot{\mathfrak{q}}_x$, $\dot{\mathfrak{q}}_y$, $\dot{\mathfrak{q}}_z$ we consider as infinitely small; *i.e.* besides magnitudes that only contain one of these components as factor, we neglect terms in which two such factors occur.

Any of the considered bodies shall be homogeneous. However, for that the cases of reflexion and refraction are not excluded, we imagine two different bodies, they may (Fig. 1) either sharply mutually separated at a surface Σ , or steadily go into one another at a thin limiting layer, such as between the surfaces Σ_1 , and Σ_2 , (Fig. 2). If we speak in the latter case about a "limiting surface", then we shall mean by that, for example, a surface Σ halfway between Σ_1 and Σ_2 .

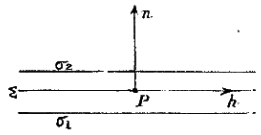

Fig. 1.

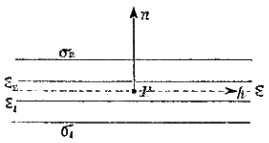

Fig. 2.

We will always calculate by averages, and namely not only by those defined in §4 *l*, but sometimes also by others, that come into consideration when the relevant magnitude only exist in a single point *Q*, for example in one point of the various molecules, or if we have reason to consider only the values of a function in such points. Such a average of *second* kind we distinguish from the averages of *first* kind by a double horizontal prime, and besides we follow a similar calculation rule as during the last calculation. Namely we understand under the value of $\overline{\overline{\phi}}$ in a point *P* the arithmetic average of the values of φ in the points Q, so far as they are present within the sphere *I* around *P* (as mentioned in §4, *l*).

By the assumption made about the radius *R* (§4), all "rapid" variations are vanished from the averages; however, concerning the velocity of the remaining variations, we have to distinguish between the interior of the body and the border. If we are positioning in Figures 1 and 2 the surfaces σ_1 and σ_2 in such a way, so that in the first figure they are both distant from Σ by *R*, while in the second this distance exists, first, between Σ_1 and σ_1 , and second, between Σ_2 and σ_2 , then for the calculation of $\overline{\overline{\phi}}$ or $\overline{\phi}$ in the points, that are outside of the layer (σ_1 , σ_2), only the values of φ come into play. While the averages, although in a completely steady way, can be considerably different from σ_1 to σ_2 , we want to assume, that the variations from point to point are much slower in the interior of the body. This will be indeed satisfied in the problems to be considered, when only the wavelength λ is many times greater than the distance *a* of σ_1 and σ_2

We even want to assume, that between λ and *a*, we can also introduce the distance *l*, so that λ/l and l/a become very great. The purpose of this assumption will become clear soon.

If the limiting surface Σ is curved, then the radii of curvature shall be greater than λ , or at least of the same order.

§ 40. We already have spoken about the electric moment of a molecule in § 33. We want also now to retain the definition given there, and in similar manner call the vector

where the sum is extended over all ions in the interior of sphere *I*, the *moment of unit volume*. More precisely we say, that (47) may indicate the value of this moment in the center of the sphere. If we choose for this new vector the sign $\mathfrak{M}$, then

With this $\mathfrak{M}$, another magnitude is most closely connected. During the displacement of the ions from the equilibrium positions, a fixed surface will namely be interspersed, which we may call a "convection current through the surface". If $d\sigma$ is an element of surface, with *P* as its center and *n* as its perpendicular, then the charge

ϵ that passed through it into the side designated by *n*, will depend on the location of *P*, if we specify the magnitude $d\sigma$ and the direction of *n* once and for all. Let $d\sigma$ be very small in relation to the molecular distances, but as great, so that we don't have to consider the cases in which an ion just is in contact with the borderline. Obviously some locations of *P* will exist, where the element won't intercept any ion at all, and others where it will intersect the path $\mathfrak{q}$ of an ion. In the first case $\epsilon = 0$, in the latter it is equal to the positive and negative calculated charge of the ion.

Since ϵ depends on the location of *P*, we can form the average $\overline{\epsilon}$ in the ordinary way; it is now, as it shall be shown in the next §,

$$\mathfrak{M}_n d\sigma \, .$$

§ 41. The rule contained in the formula

$$\overline{\epsilon} = \tfrac{1}{l} \int \epsilon d\mathfrak{r}$$

can be express somewhat differently. Namely we shall choose for the point *P* an infinite amount, we want to say *k*, locations that are uniformly distributed over the sphere *I*, and take the arithmetic mean of the values of ϵ that are valid for these locations, *i.e.* we put

Any ion, that has its equilibrium position in the interior of *I*, will (during its displacement) now pass through some positions that are connected with the element $d\sigma$ and thus add some terms to the sum $\Sigma\epsilon$. We obtain the whole sum, if we at first add to one another the terms that stem from a certain ion, and than sum over all ions.

Let *Q* be the equilibrium position of the considered ions, and *Q* the new position; so $QQ' = \mathfrak{q}$. The length and the direction of this line are given, as well as the direction and magnitude of $d\sigma$. Whether the particle hits the surface element and provides the part *e* for the sought sum, only depends on the relative positions of *P* and *Q*. Thus we can, instead of giving *k* positions in the sphere *l* to *P*, also let remain the point at its position and lead point *Q* around a sphere *I*. As QQ' now hits the fixed element $d\sigma$, when *Q* lies in a certain, easily specifiable cylinder of area $\mathfrak{q}_n d\sigma$, then the number of "relevant" positions is related to the integer *k*, as the area of that cylinder is related to the area of sphere *I*. This number is thus

$$\tfrac{k}{I} \mathfrak{q}_n d\sigma \, ,$$

and the sum $\Sigma\epsilon$, as far it is caused by the ion *Q*,

$$\tfrac{k}{I} e \mathfrak{q}_n d\sigma \, .$$

Eventually in formula (49) we obtain

$$\Sigma\epsilon = \tfrac{k}{I} \Sigma e \mathfrak{q}_n \cdot d\sigma \, ,$$

where the sum is extended over all ions of sphere *I*, and

$$\overline{\epsilon} = \tfrac{k}{I} \Sigma e q_n \cdot d\sigma \, ,$$

or by (48)

$$\overline{\epsilon} = \mathfrak{M}_n d\sigma \, .$$

§ 42. Equations (I_b) — (VII_b) (§ 20) may form the initial point for the subsequent considerations. At first we notice, that the first of them is equivalent to

$$\int \mathfrak{d}_n d\sigma = E \, ,$$

for an arbitrary closed surface (*n* is to be drawn into the outside), when *E* is the electric charge enclosed by it. If now in one element $d\mathfrak{r}$ of the inner space in equilibrium state, a density ρ_0 exists, and if (for an element of the surface) ϵ has the meaning given above, then

$$E = \int \rho_0 d\mathfrak{r} - \Sigma\epsilon \, ,$$

where the sum is related to all elements $d\sigma$.

By that

$$\int \mathfrak{d}_n d\sigma + \Sigma\epsilon = \int \rho_0 d\mathfrak{r} \, .$$

From the definition of the average we easily find now

$$\int \overline{\mathfrak{d}_n} d\sigma + \Sigma\overline{\epsilon} = \int \overline{\rho_0} d\mathfrak{r} \, .$$

Since now

$$\overline{\rho_0} = 0 \, ,$$

and

$$\overline{\epsilon} = \mathfrak{M}_n d\sigma \, .$$

it is eventually given

$$\int \left(\overline{\mathfrak{d}_n} + \mathfrak{M}_n \right) d\sigma = 0 \, .$$

We now want to define a new vector $\mathfrak{D}$ by the equation

$$\mathfrak{D} = \overline{\mathfrak{d}} + \mathfrak{M} \, ,$$

and call it the *dielectric polarization*.

This vector, that goes over for the free aether (where $\mathfrak{M} = 0$) into $\mathfrak{d}$, is exactly that, what Maxwell calls "dielectric displacement". Its basic property is according to the above, that for any closed surface

and also in the interior of any body

§ 43. Formula (50) leads to an important limiting-condition, if we apply it to a surface, that lies partly in

the first, and partly in the second body. Around a certain point *P* of the limiting-surface Σ (Fig. 1 and 2) we shall lay a cylinder-surface *C* that is parallel to the perpendicular in *P*, and choose for the mentioned area the surface of the space that is cut from layer (σ_1 , σ_2). If now the dimensions of the parts limited in σ_1 and σ_2 are of order *l* (§ 39), then we may consider the parts as elements that are equal, parallel and plane, and as they are much greater than the part of *C* that lies between σ_1 and σ_2 , we can omit the integral taken over the latter

$$\int \mathfrak{D}_n d\sigma$$

Thus we find, if we mutually distinguish the values that are valid in σ_1 and σ_2 by the indices 1 and 2, and draw either at σ_1 as well as at σ_2 the perpendicular *n* from the first to the second body,

In relation to this, we have to notice one thing. In any medium, $\mathfrak{D}_x, \mathfrak{D}_y, \mathfrak{D}_z$ can be represented as slowly (§ 39) varying functions of coordinates, and we would have to substitute in these functions the coordinates of a point of σ_1 or σ_2 , to obtain $\mathfrak{D}_{n(1)}$ and $\mathfrak{D}_{n(2)}$. Instead of this we can without noticeable error — due to the small distance of the surfaces — introduce the coordinates of the point *P* that lies in Σ . Thus it is allowed to say, that $\mathfrak{D}_{n(1)}$ and $\mathfrak{D}_{n(2)}$ are the values *at the limiting-surface* and that the previous formula *expresses the continuity of* $\mathfrak{D}_n$.

Similar formulas as equations (I_c) and (51) are emerging from (II_b); namely for the interior of a body

$$Div\, \overline{\mathfrak{H}} = 0\,,$$

and for the limiting-surface

$$\overline{\mathfrak{H}}_{n(1)} = \overline{\mathfrak{H}}_{n(2)}\,.$$

§ 44. From fundamental equation (III_b) we derive

$$Rot\, \overline{\mathfrak{H}'} = 4\pi\overline{\rho\mathfrak{v}} + 4\pi\dot{\overline{\mathfrak{d}}}\,,$$

or, be means of the definition

This derivation is true for the interior of a body. To arrive at the limiting condition, we note at first, that (§ 4, *h*) (by the equation (III_b) for an arbitrary surface σ , with the borderline *s')*

$$\int \mathfrak{H}_s' ds = 4\pi \int \left(\rho\mathfrak{v}_n + \dot{\mathfrak{d}}_n\right) d\sigma$$

and thus also

Now we lay through the point *p* (Fig. 1 and 2) a plane, that contains the perpendicular of the borderline and the arbitrary direction *h* tangential to Σ , and choose as surface σ the part of this plane, that lies between σ_1 and σ_2 and which is limited by two lines parallel to that perpendicular. If the length of this layer in the direction *h* is of order *l* (§ 39), then we may neglect all magnitudes of order *a* and we obtain from (52)

$$\overline{\mathfrak{H}'}_{h(1)} = \overline{\mathfrak{H}'}_{h(2)}\,.$$

where the indices 1 and 2 have the same meaning as above. For the two components of $\overline{\mathfrak{H}'}$ we may take at this place the values in point *P* again, and thus the equations says, that the *tangential components of vector* $\overline{\mathfrak{H}'}$ *were steady*.

§ 45. Equation (IV_b) admits of a similar application. Before, I give the remark that no magnetic forces exists, as long the ions are at rest, thus that $\mathfrak{H}$ is of same order as the velocities $\mathfrak{v}$. In (VII_b) we can consequently neglect the last term; it becomes $\mathfrak{F} = \mathfrak{E}$, consequently by (IV_b) for the interior of a body

$$Rot\, \overline{\mathfrak{E}} = -\dot{\overline{\mathfrak{H}}}\,,$$

and for the borderline

$$\overline{\mathfrak{E}}_{h(1)} = \overline{\mathfrak{E}}_{h(2)}\,.$$

At last it still follows from (V_b) and (VI_b)

and

1.2 The equations of motion for ions.

§ 46. So far everything was quite simple. Yet great difficulties arise, when we also want to form the equations of motion for the oscillating ions themselves. To express in these equations the relations, which are the basis for dispersion, birefringence, and circular polarization, it would be required an understanding of molecular processes that wasn't achieved by us by far. We want to restrict ourselves, to derive from a very simple presupposition the most probable shape of the sought relations, and then help on ourselves as good as possible. It is of course an advantage, that for this task we have to consider the interior of the homogeneous body, since (regarding the borderlines) the already derived equations enclose all required conditions.

The mentioned assumption is now, that any of the mutual completely equal molecules, only contains a single movable ion, while all others are fixed.

Let *m* be the mass of a movable ion, $\mathfrak{K}$ the total force acting on it, *N* the number of molecules in unit volume. For the equations

$$m\frac{d^2\mathfrak{q}_x}{dt^2} = \mathfrak{K}_x\,, \text{ etc.}$$

it follows, when we take the averages of second kind and multiply them by *eN*

$$m\frac{\partial^2\mathfrak{M}_x}{\partial t^2} = eN\overline{\overline{\mathfrak{K}_x}}\,, \text{ etc.}$$

As regards $\mathfrak{K}$, it is at first to note, that by our assumption the fixed parts of the molecule are acting upon the ion by a certain force, that is exactly caused by the displacement $\mathfrak{q}$. Let the components of this force be linear, homogeneous functions of $\mathfrak{q}_x, \mathfrak{q}_y, \mathfrak{q}_z$, or rather, since only this is relevant for the following, let the averages of those components be given by

in which certain constants are denoted by *s*.

We also assume for these forces, that they won't be changed by the translation $\mathfrak{p}$, at least not as regards magnitudes of first order.

§ 47. In consequence of the electric motions, also the aether exerts an action upon the ions. This can be derived from formula (V_b), since $\mathfrak{E} = \mathfrak{F}$ as we have seen. If it would be allowed, to put for the electric force $\mathfrak{E}$ everywhere the average $\overline{\mathfrak{E}}$, that has the same magnitude and direction at all points of an ion, then we would have to add into the expressions (55) only the terms

But this matter isn't all that simple. First, the oscillating ion itself causes a value of $\mathfrak{E}$, that is not the same in all points of the particle, so that we could find the corresponding part of $\mathfrak{K}$ only by an integration over the space in which the ion is located. Second, even if we could neglect this, for the calculation of $\overline{\overline{\mathfrak{K}}}$ the average $\overline{\overline{\mathfrak{E}}}$ is of relevance, not the average $\overline{\mathfrak{E}}$, and it is not allowed, to mutually interchange both. Of course nothing would be in the way, in so far the motions of ions that cause the electric force $\mathfrak{E}$, take place in the distance *P* from the considered point that is much greater as the distance of the molecules, but $\mathfrak{E}$ is partly caused by molecules that are located more nearly — we want to say, by the oscillations within the sphere *I* drawn around *P* — and an inequality $\overline{\mathfrak{E}}$ and $\overline{\overline{\mathfrak{E}}}$ is very well possible for a irregular distribution of the thus produced states in the aether.

When we now, in agreement with these remarks and to obtain $\overline{\overline{\mathfrak{K}}}$, add to the expressions (55) not only the values (56) but also certain supplementary terms

$$\mathfrak{k}_x,\ \mathfrak{k}_y,\ \mathfrak{k}_z$$

and thus put

then we can maintain for the magnitudes $\mathfrak{k}$, that they only depend on processes within sphere *I*. Additionally it is given, that also the supplementary terms only exist during the displacement of the ions from their equilibrium positions and — since $\mathfrak{q}$ can be considered as infinitely small — they must be linear, homogeneous functions of the magnitudes $\mathfrak{q}, \dot{\mathfrak{q}}$, etc., or rather of their averages. In consequence of equations (48), also $\mathfrak{k}$ are homogeneous, linear functions of the values of $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z, \dot{\mathfrak{M}}_x$, etc. in the various points of the spherical space *I*. Eventually we still have to consider, that all these values can be expressed by application of Taylor's theorem by the values, which will be assumed by $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z, \dot{\mathfrak{M}}_x$, etc., and the derivatives with respect to *x, y, z* in the considered point *P*, the center of the sphere. All these values thus are linearly included into the expressions for $\mathfrak{k}_x, \mathfrak{k}_y, \mathfrak{k}_z$.

To which extend these latter ones must contain the translation velocity $\mathfrak{p}$, remains undecided for now. In any case, since we neglect magnitudes of second order, only the first powers of $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$ will occur. If we also consider now, that in formulas (57) the magnitudes $eN\overline{\overline{\mathfrak{q}_x}}$, etc., could be replaced by $\mathfrak{M}_x$, etc., and if we think of these equations as solved with respect to $\overline{\mathfrak{E}_x}$, etc., then we can see, that these components of the electric force can be represented as linear, homogeneous functions of $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z$ and their derivatives with respect *x, y, z, t*, and that the coefficients in these functions can linearly contain the velocities $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$.

For brevity, the equations that would result in a completely developed theory for $\overline{\mathfrak{E}_x}, \overline{\mathfrak{E}_y}, \overline{\mathfrak{E}_z}$, may be summarized in the formula

As regards any of the vectors $\mathfrak{M}, \dot{\mathfrak{M}}, \ddot{\mathfrak{M}}$, ..., we also have to consider the derivatives of its components with respect to the coordinates.

If we now eventually let fall our simplifying presupposition, and consider any molecule as a formation of, maybe, very complicated structure that contains several movable ions, then it is near at hand to assume, that still a relation like the one represented in (58) does exist. Our next task shall be, to simplify as much as possible the relation by means of certain, general considerations.

1.3 Simplification for transparent bodies.

§ 48. If a certain motion exists in a system, then, as it was shown in § 18, also the inverse motion is possible, as soon as forces of non-electric origin are the same for a certain location of the ions as well, as in the original case. From this it directly follows, that all motions in a body, that besides ions also contain uncharged mass particles, can be reversed, in case all molecular forces are determined by the configurations and not, for example, depending on the velocities.

During the inversion of motions all velocities obtain an opposite direction, thus also the translation $\mathfrak{p}$. Furthermore we can easily see, — look at formulas of §§ 43 and 44 —, that in the new state at time *t*, the vectors

$$\mathfrak{M},\ \overline{\mathfrak{H}} \text{ und } \overline{\mathfrak{E}}$$

have the same direction and magnitude, as the vectors

$$\mathfrak{M},\ -\overline{\mathfrak{H}} \text{ und } \overline{\mathfrak{E}}$$

in the original state at time *-t*.

Obviously, the *transparent* bodies, namely only those,[1] in which the light motions are reversible in the alluded sense, and it may be clearly emphasized, that the circular polarizing substances form no exception from this rule.[2]

We now want to see, which simplification of equation (58) is obtained from this reversibility; there, terms without and with $\mathfrak{p}$ shall be considered separately.

§ 49. If $\mathfrak{p} = 0$, then it must be possible to express $\overline{\mathfrak{E}}_x, \overline{\mathfrak{E}}_y, \overline{\mathfrak{E}}_z$ as homogeneous, linear functions of the magnitudes $\mathfrak{M}_x, \dot{\mathfrak{M}}_x, \ddot{\mathfrak{M}}_x$, etc., and their derivatives with respect to the coordinates; the relations that serve for this, must stay unchanged, when we pass to the inverse motion. As to this motion we now have (at time *t*) $\overline{\mathfrak{E}}_x, \overline{\mathfrak{E}}_y, \overline{\mathfrak{E}}_z$, and also the components $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z$, as well as their derivatives with respect to the coordinates, have the same value and the same sign as with the original motion (at time -*t*). The same is true for all *even* derivatives with respect to time. The *uneven* derivatives with respect to *t* have, however, the same magnitude as regards the two motions, but opposite signs, and thus these derivatives cannot occur in the relations between $\overline{\mathfrak{E}}$ and $\mathfrak{M}$. To indicate this, we replace (58) for resting bodies by

If we again allow the translation, then we have to add to F_1 still another vector, whose components are linear and homogeneous functions of $\mathfrak{M}, \dot{\mathfrak{M}}, \ddot{\mathfrak{M}}$, ..., and which contain in any term one of the factors $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$; also this new vector must stay unchanged when passing to the inverse motion. As in this case the components $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$ contain opposite signs, thus they can only be multiplied by such magnitudes which also change the sign, *i.e.* by uneven derivatives with respect to time. The equations (58) therefore generally assume the form

An additional simplifications we can achieved, by considering a certain kind of homogeneous light, *i.e.* by considering goniometric functions of time of a certain period *T*. Then

If we in (60) express in this way all even derivatives by $\mathfrak{M}$ and all uneven by $\dot{\mathfrak{M}}$, it will be given

The components of F_1 are now homogeneous functions of $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z$ and its derivatives with respect to *x, y, z*, while F_2 depends in a similar way on $\dot{\mathfrak{M}}$. The coefficients of this function may well depend on the oscillation period *T*, since we have introduced the values (61) into (60).

1.4 The dispersion of light.

§ 50. There are two ways of attempting to explain the dispersion of colors, either by (like Cauchy) considering from location to location the variation of the equilibrium disturbance, or by considering as relevant the variation with respect to time. In one case it is the wave length, in the other one the oscillation period, that *directly* determines the propagation velocity, although at the end both have the same result.

If we would take the first path and also reproduce the explanation given by Cauchy — in its mathematical form — in our theory, then we would have to assume, that the equations summarized in (59) likely contain derivatives with respect to *x, y, z*, but not such with respect to *t*, and that namely, due to the smallness of *m*, the first term in (57) would vanishes. It is clear, that the propagation velocity must change with wave length, as soon as, for example, $\mathfrak{M}_x$, and $\frac{\partial^2 \mathfrak{M}_x}{\partial y^2}$ are standing next to one another. Namely, the latter magnitude gains with respect to the first a greater influence, the smaller the wavelength.

The straight opposite assumption would be, that only derivatives with respect to *t*, but none with respect to *x, y, z* occur in formula (59). Now, in so far, that the only magnitude of the first kind (whose introduction has proven to be necessary) is the term

$$m \frac{\partial^2 \mathfrak{M}_x}{\partial t^2}$$

in equation (57), we can say that the second mentioned view reduces the phenomenon to the mass of the co-oscillating ion.

That this explanation can really be *achieved* now, was already proven by v. Helmholtz and earlier also by me. The new form that I now give to the theory, makes no difference in this respect.

As we know, mainly the phenomena of anomalous dispersion speak in favor of the assumption of co-oscillating masses. On the other hand, as regards the derivative with respect to *x, y, z*, it is the question, whether the terms in which they occur, are really great enough to exert a considerable influence. As we saw, the mentioned terms can only stem from the fact, that the electric moment $\mathfrak{M}$ doesn't have in all points of sphere *I* the same magnitude and direction. Since the radius is much smaller than the wave length, thus the differences are surely very insignificant, and we won't hesitate to neglect them, if it is about an action upon a distant point. Anyway, is would be premature to claim that also this small variation of $\mathfrak{M}$ couldn't have an influence on the phenomena in the interior of the sphere. The rotation of the polarization plane, to which we will return too, which presumable can't be understood without the aid of derivatives with respect to *x, y, z*, must prevent us from denying from the outset an influence of such terms on dispersion.

With more justification we can derive *from the phenomena* the insignificance of that influence. Namely, if we retain in equations (59) the derivatives with respect to *x, y, z*, and then simplify the equations, so far it is possible due to the known symmetry relations of crystals, then we are lead to laws for the motion of light, which are more complicated than the ones actually applied, and only go over into them by further simplification of the formulas, for which we cannot give any reason. For example, according to these laws the regular crystals wouldn't be isotropic, but must show a peculiar kind of birefringence.[3]

The things said may justify, that we, while preliminarily the circular-polarizing media remain excluded, assume for the other transparent bodies that the relation (62) contains no derivative with respect to *x, y, z*. We thus put

and here we understand by $(\dot{\mathfrak{M}}, \mathfrak{p})_x$, $(\dot{\mathfrak{M}}, \mathfrak{p})_y$, $(\dot{\mathfrak{M}}, \mathfrak{p})_z$

expressions, which are linear and homogeneous with respect to $\dot{\mathfrak{M}}_x,\ \dot{\mathfrak{M}}_y,\ \dot{\mathfrak{M}}_z$ as well as to $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$. The coefficients in these expressions, as well as the factors σ are to be viewed as functions of *T*.

Now I will prove, that for a very general class of bodies, the terms $(\dot{\mathfrak{M}}, \mathfrak{p})_x$, etc. will vanish; at the same time we reach on that occasion also a simplification of the terms independent from $\mathfrak{p}$.

1.5 Bodies with three mutually perpendicular planes of symmetry.

§ 51. Let *A* be an arbitrary body, and A' a second body that is the mirror image of the first one with respect to a certain plane *E*, and namely down to the smallest parts, thus also for the distribution of the smallest particles. If the molecular forces depend in such way from the configurations, that the vectors, by which they were represented in *A* and A' , behave like objects and their mirror images, then ion motions can occur in the two bodies in connection with state changes of aether (§ 18), so that also regarding these phenomena, one system is forever the mirror image of the other one. When passing from the first system to the second, the vectors $\overline{\mathfrak{E}}, \mathfrak{M}$ and $\mathfrak{p}$ are transformed into their mirror images.

The inner construction of body *A* can only be such, by appropriate choice of the plane *E,* so that *A* and A' with respect to the *same* coordinate system have the same properties, *i.e.* that the properties in *A* and A' can be expressed by the *same* equations, without change of a constant or sign. In this case we call *E* a *plane of symmetry*. The bodies which we now consider, and to which we will restrict ourselves preliminarily, are those, for which three mutually perpendicular symmetry planes of this kind exist.

We give the coordinate planes the direction with of the symmetry-planes, and consider at first the mirror image with respect to the *yz*-plane. When passing to this image, $\overline{\mathfrak{E}}_x, \mathfrak{M}_x$ and $\mathfrak{p}_x$ change their sign, while the other components of $\overline{\mathfrak{E}}, \mathfrak{M}$ and $\mathfrak{p}$ remain completely unchanged. This is only possible, when (after $(\dot{\mathfrak{M}}, \mathfrak{p})_x$, etc. are represented as functions of $\dot{\mathfrak{M}}_x,\ \dot{\mathfrak{M}}_y,\ \dot{\mathfrak{M}}_z,\ \mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$) the index *x* appears in every term of the first formula once, or in every term of the second and third either not at all, or two times. To a similar conclusion we come also with respect to indices *y* and *z*. If we additionally consider the mirror images with respect to the *zx*- and the *xy*-plane, then we find, that not a single term as $(\dot{\mathfrak{M}}, \mathfrak{p})_x$ is applicable, and that from the nine coefficients σ , only $\sigma_{1.1}$, $\sigma_{2.2}$ and $\sigma_{3.3}$ can be different from zero.

Thus we obtain

or

$$\tfrac{4\pi V^2}{\sigma_{1.1}}\overline{\mathfrak{E}}_x = 4\pi V^2 \mathfrak{M}_x \text{ , u.s.w.}$$

If we add these formulas to the three summarized in (53), and put

$$1 + \tfrac{4\pi V^2}{\sigma_{1.1}} = \varkappa_1,\ 1 + \tfrac{4\pi V^2}{\sigma_{2.2}} = \varkappa_2,\ 1 + \tfrac{4\pi V^2}{\sigma_{3.3}} = \varkappa_3,$$

then

$$\varkappa_1 \overline{\mathfrak{E}}_x = 4\pi V^2 \mathfrak{D}_x + [\mathfrak{p}.\mathfrak{H}]_x$$

where, for a certain type of light, $\varkappa_1, \varkappa_1$ and $\varkappa_3$ are constants.

1.6 Summary of the equations.

§ 52. Neglecting the primes over the letters — since was continue to only speak about averages — we summarize the equations of motion now in the following way.

In the interior of any body it is given

and

since, neglecting magnitudes of second order, we may replace, by the relation (53), $4\pi\overline{\mathfrak{d}}$ by $\mathfrak{E}/V^2$ in equation (54).

At the borderline the conditions apply

If there is no translation, then $\mathfrak{H}'$ falls into $\mathfrak{H}$; then the equations (III_c) and (V_c) go over into

and the last of the limiting conditions ($VIII_c$) into

$$\mathfrak{H}_{h(1)} = \mathfrak{H}_{h(2)} \ .$$

Thus for this case, the known equations of motion and limiting conditions of the electromagnetic theory of light are given. From formulas $(I_c), (II_c), (III'_c), (IV_c)$ and (V'_c) we derive (when $\varkappa_1, \varkappa_2, \varkappa_3$ are different from each other) the laws of light motion in crystals of two axes, while the assumption $\varkappa_1 = \varkappa_2 = \varkappa_3$ leads back to isotropic bodies. Besides, since $\varkappa_1, \varkappa_2$ and $\varkappa_3$ depend on the oscillation period, also the explanation of the dispersion of light is contained in the formulas.

Also the case of the pure aether is not excluded. Since no electric moments $\mathfrak{M}$ exist in it, then we have to set by (64) $\sigma_{1.1} = \sigma_{2.2} = \sigma_{3.3} = \infty$ and thus $\varkappa_1 = \varkappa_2 = \varkappa_3 = 1$. The equations V_c) and (V'_c) thereby are transformed into

We can easily see, that the equations which we obtain in this way for the aether, are in agreement with formulas (I)-(V) or (I_b)-(VII_b)

It is self-evident, as regards the interior of the pure aether, that the connection between the various magnitudes is always the same, the ponderable matter may be in motion or not.

1.7 Circular polarizing media.

§ 53. Bodies, which turn the polarization plane, were excluded above. It's not feasible to form a thorough theory

for them until now; nevertheless some general consideration, as required by our purpose, may find their place here.

Since the rotation of the polarization plane is connected with the fact, that the medium is not in accordance in all its properties with its mirror image, then the things said in § 51 are not applicable anymore. Nevertheless, everything becomes quite easy when we restrict ourselves to *isotropic* media.

If we assume, in the relation between $\overline{\mathfrak{E}}$ and $\mathfrak{M}$, that no derivatives with respect to *x, y, z* are present, then we have to understand under $F_1(\mathfrak{M})$ of equation (62), a vector that is completely determined even by $\mathfrak{M}$, namely the isotropy requires that the figure consisting of $\mathfrak{M}$ and $F_1(\mathfrak{M})$ can be rotated in an arbitrary manner, without that $F_1(\mathfrak{M})$ ceases to fit to $\mathfrak{M}$. If we now choose the direction of $\mathfrak{M}$ itself as the rotation axis, then $\mathfrak{M}$ always remains the same vector; thus $F_1(\mathfrak{M})$ must remain unchanged, which is only possible when this vector has the direction of $\mathfrak{M}$. With respect to the linear character of the sought relation, we consequently have to set

where σ is a scalar constant.

The second vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$ occurring in (62), has the following properties. First, its components are homogeneous, linear functions of $\dot{\mathfrak{M}}_x$, $\dot{\mathfrak{M}}_y$, $\dot{\mathfrak{M}}_z$ as well as from $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$. Second, after an arbitrary rotation of the figure consisting of the three vectors $\dot{\mathfrak{M}}, \mathfrak{p}$ and $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$, $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$ must still fit to $\dot{\mathfrak{M}}$ and $\mathfrak{p}$. By that we derive[4]

where *k* is a positive or negative constant, which by the way, as σ above, can also depend on the oscillation period *T*.

§ 54. The presupposition, that no derivatives with respect to *x, y, z* occur, has led us to equation (65), from which the rotation of the polarization plane does *not* arise. Thus it is necessary, as it was already indicated earlier to assume (at least in the expression $F_1(\mathfrak{M})$) derivatives with respect to the coordinates. The most simple is, to add to the second term of (65) another vector $\mathfrak{R}$, whose components do linearly and homogeneously depend on the *first* derivatives of $\mathfrak{M}_x$, $\mathfrak{M}_y$, $\mathfrak{M}_z$. Magnitude and direction will now again be closely determined by isotropy. Namely, if we imagine at any point of space a line, that represents the vector $\mathfrak{M}$, and in addition in the considered point the vector $\mathfrak{R}$, then after an arbitrary rotation of that entire figure, $\mathfrak{R}$ must still fit to $\mathfrak{M}$. Only the assumption[5]

$$\mathfrak{R} = j\ Rot\ \mathfrak{M}\ ,$$

is in agreement with this, where *j* is a certain constant and which we want to add for resting bodies (65) to

$$F_1(\mathfrak{M}) = \sigma\mathfrak{M} + j\ Rot\ \mathfrak{M}$$

Now, we could introduce (into the term $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$) derivatives with respect to *x, y, z*; however, we will omit this, since the things already said are sufficient for our purpose. By that we have (when we omit the prime over $\mathfrak{E}$ from now on) to put for isotropic, circular-polarizing media

§ 55. It is not without interest, to consider for a moment the mirror image of a motion to which the found equation applies. The vectors that apply to this new motion, which may be called $\mathfrak{E}', \mathfrak{M}', \dot{\mathfrak{M}}'$ and $\mathfrak{p}'$, are mirror images of the vectors $\mathfrak{E}, \mathfrak{M}, \dot{\mathfrak{M}}$ and $\mathfrak{p}$. From that if follows, that the mirror images of $Rot\ \mathfrak{M}$ and $[\dot{\mathfrak{M}}.\mathfrak{p}]$ don't fall into $Rot\ \mathfrak{M}'$ and $[\dot{\mathfrak{M}}'.\mathfrak{p}']$, but into $-Rot\ \mathfrak{M}'$ and $-[\dot{\mathfrak{M}}'.\mathfrak{p}']$. Now, since the linear relation between four vectors expressed in (68), also then remains when we replace any of them by its mirror image, hence

$$\mathfrak{E}' = \sigma\mathfrak{M}' - j\ Rot\ \mathfrak{M}' - k[\dot{\mathfrak{M}}'.\mathfrak{p}']$$

By that we see, that the processes that can occur in the mirror image of the considered body, don't satisfy the relation (68) anymore, but a relation in which the terms with *j* and *k* have difference signs. Thus it is confirmed, that these terms are likely be connected with the fact, that body and its mirror image have different properties; we may expect, that a rotation of the polarization plane will actually be in agreement with them.

I postpone the details about this. Here, it only shall be remarked that the magnitude $j\ Rot\ \mathfrak{M}$ (we will make that the natural rotation of the polarization plane will depend on it) has much similarity with the terms, that were assumed by various physicists in the equations of motion of light, to explain circular-polarization. Indeed I regard, in the absence of a theory that explains the phenomenon more deeply, the introduction of the term $j\ Rot\ \mathfrak{M}$ as neither better nor worse than the hypotheses of those physicists.

The last term in (68) has a peculiar meaning. Namely a rotation of the polarization plane would correspond to it, that would be caused in a body (that is different from its mirror image) by the motion of earth[6].

[1] If we would reverse the motions in an *absorbing* medium, then a state would arise, at which the amplitude would be increased in the direction of propagation.

[2] The *magnetic* rotation of the polarization plane remains excluded from our considerations

[3] See my earlier considerations (Over het verband tauchen de voortplantingsanelheid van het licht en de dichtheid en samenstelling der middenstoffen. Verhandelingen der Akad. van Wet. te Amsterdam, Deel 18, pp. 68—77; Wied. i., Bd. 9, p. 656).

[4] If we decompose $\mathfrak{p}$ into two components $\mathfrak{p}_1$ and $\mathfrak{p}_2$, then it follows from the first mentioned property of $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$

$$F_2(\dot{\mathfrak{M}}, \mathfrak{p}) = F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1) + F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$$

It is assumed, that $\mathfrak{p}_1$ falls into the direction of $\dot{\mathfrak{M}}$, and $\mathfrak{p}_2$ is perpendicular to it. If we now rotate the figure (consisting of $\dot{\mathfrak{M}}, \mathfrak{p}_1$ and $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1)$) around an axis that falls

into $\dot{\mathfrak{M}}$, $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$ stay were they are, and thus $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1)$ may not change as well. Consequently, this vector must have the direction of $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$. That can be shown in addition, by means of a rotation of 180° around an axis perpendicular to $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$. In the course of this rotation, the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1)$ would obtain the opposite direction; yet it shouldn't be changing, because *both* vectors $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$ change their sign. To find out the direction of $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$, we turn the figure (which is formed by this vector with $\dot{\mathfrak{M}}$ and $\mathfrak{p}_2$) around an axis perpendicular to the plane $(\dot{\mathfrak{M}}, \mathfrak{p}_2)$ or $(\dot{\mathfrak{M}}, \mathfrak{p})$, namely around 180°. Here, $\dot{\mathfrak{M}}$ and $\mathfrak{p}_2$ go over into $-\dot{\mathfrak{M}}$ and $-\mathfrak{p}_2$; the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$ thus may not be changed, which is only possible when it has the direction of the axis. Thus the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$ — and thus by (67) also the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$ — is perpendicular to the plane $(\dot{\mathfrak{M}}, \mathfrak{p})$; its magnitude is proportional to the values of $\dot{\mathfrak{M}}$ and $\mathfrak{p}_2$. Both we have expressed in (66).

[5] After a rotation of the mentioned figure we want, as we are really free to do this, to apply again the *original* coordinate axis for the decomposition of the vectors and the formation of the derivatives. At first, only a rotation of 180° around the axis takes place. Here, $\mathfrak{R}_x$ remains unchanged; consequently in the expression for this component only these derivatives of $\mathfrak{M}_x$, $\mathfrak{M}_y$, $\mathfrak{M}_z$ can occur, which do *not* change the sign. These are

$$\frac{\partial \mathfrak{M}_x}{\partial x}, \ \frac{\partial \mathfrak{M}_y}{\partial y}, \ \frac{\partial \mathfrak{M}_y}{\partial z}, \ \frac{\partial \mathfrak{M}_z}{\partial y}, \ \frac{\partial \mathfrak{M}_z}{\partial z} \ .$$

If we further notice, that in the course of a rotation of 180° around the *y*- or *z*-axis, $\mathfrak{R}_x$ assumes the opposite direction, and that also those derivatives are excluded, which retain the same sign during one of these rotations, the we find, that $\mathfrak{R}_x$ must be of the form

$$j\frac{\partial \mathfrak{M}_z}{\partial y} + j\frac{\partial \mathfrak{M}_y}{\partial z}$$

Eventually we imagine still another rotation around 90° around the *x*-axis, whereby *OY* is transformed into *OZ*. After that rotation, $\frac{\partial \mathfrak{M}_z}{\partial y}$ and $\frac{\partial \mathfrak{M}_y}{\partial z}$ have the values, that previously belonged to $-\frac{\partial \mathfrak{M}_y}{\partial z}$ and $-\frac{\partial \mathfrak{M}_z}{\partial y}$; however, since $\mathfrak{R}_x$ hasn't changed, then $j' = -j$. From $\mathfrak{R}_x$ we find $\mathfrak{R}_y$ and $\mathfrak{R}_z$ by permutation of the letters.

[6] The following consideration might be sufficient, to make the existence of the electric force $k[\dot{\mathfrak{M}}.\mathfrak{p}]$ somewhat probably, for which only the possibility was shown in the text. Since a molecule of a circular-polarizing substance must have a so-called "helical" structure, then the particles from which it consists may be mutually connected, so that the displacement of one of them produces a circular motion of one or many others. Let, for example, a positive ion *A* be in motion along the line *G*, and by that the moment $\mathfrak{M}$ shall be produced, so that the velocity is proportional to $\dot{\mathfrak{M}}$, and this motion may be accompanied by the rotation (in a circle with *G* as its axis) of some other ions *B* that are also positive. Between the velocities of *A* and *B* there is a constant relation. The motion of particle *B* then forms a circular electric current, proportional to $\dot{\mathfrak{M}}$, and this produces in the molecule and in its vicinity a *local* magnetic force, which in *A* falls into line *G* and thus also into $\dot{\mathfrak{M}}$, and which is proportional to $\dot{\mathfrak{M}}$. If we combine, in accordance with the last term of fundamental equation (V), this magnetic force with the velocity $\mathfrak{p}$, then we obtain an electric force like $k[\dot{\mathfrak{M}}.\mathfrak{p}]$.

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